Main Ideas in Class Today

After today's class, you should be able to:

- Understand Simple Harmonic Motion (SHM)
- Determine the Position, Velocity and Acceleration over time
- Find the Period and Frequency of SHM
- Relate Circular Motion to SHM
- Relate SHM to the motion of a pendulum

Extra Practice: 13.17, 13.19, 13.21, 13.23, 13.25, 13.27, 13.29, 13.31, 13.33, 13.35, 13.37, 13.39, 13.41

Simple Harmonic Motion

- Any vibrating system with F proportional to $-x$ like Hooke's law (F=-kx) undergoes SHM
- System is called a simple harmonic oscillator (SHO)
  - Ex: Spring; pendulum (for small amplitudes), a car stuck in a ditch being ``rocked out", a person on a swing, vibrating strings, even sound (Ch.14)!

Main Ideas Today

How position, velocity and acceleration change with time for a simple harmonic oscillator (e.g. spring or pendulum) and how velocity changes with position and time

Comparing SHM to the Pendulum

Period and Frequency of a Spring

- Period $$T = 2\pi\sqrt{\frac{m}{k}}$$

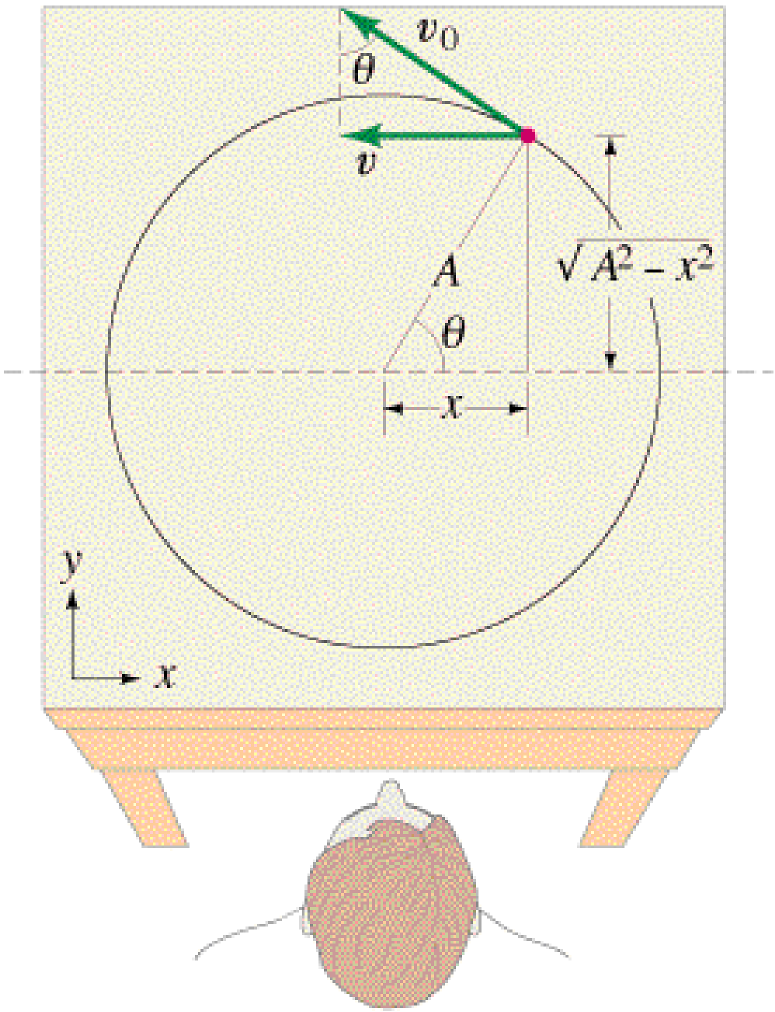
Not dependent on amplitude A!

  - The period (T) of a mass on a spring is dependent upon the mass m and the spring constant k

- Frequency $$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

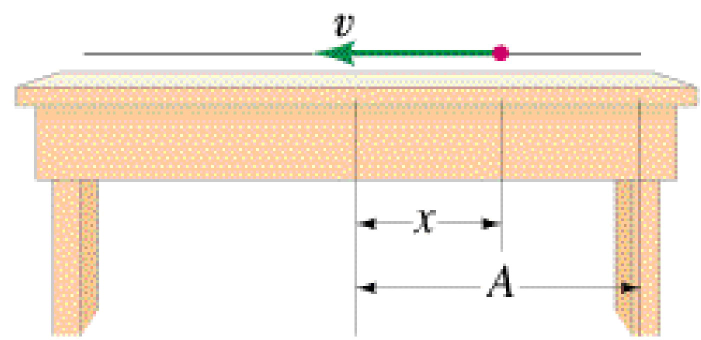
  - The frequency, f, is the number of complete cycles or vibrations per second
  - Units are s^{-1} or Hertz (Hz)

Side View of Circular Motion



(a)

$$v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$



(b)

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Motion around a circle as viewed from the side has a the same position dependence as a spring

Angular Frequency
(useful to relate to circular motion)

- The angular velocity is related to the frequency

$$\omega = 2\pi f = \sqrt{\frac{k}{m}} \qquad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

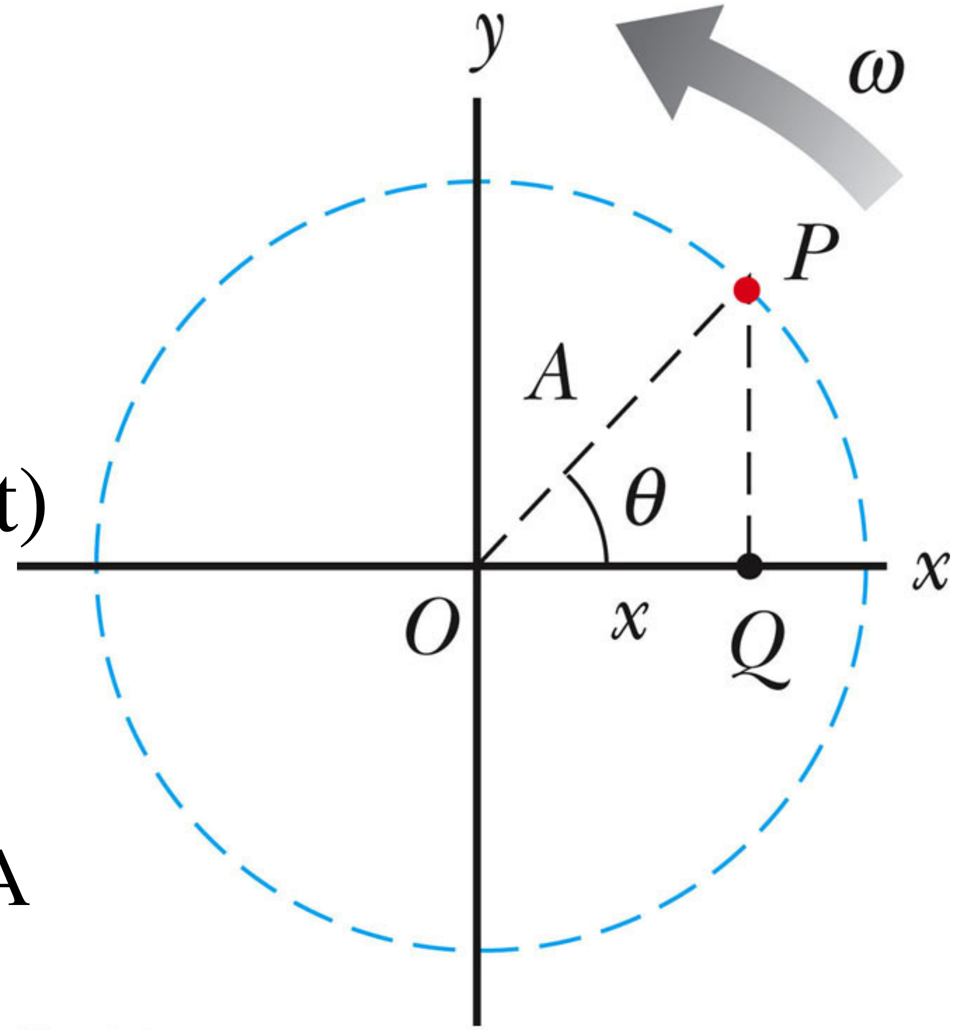
- The *frequency* gives the number of cycles per second
- The *angular velocity/speed* (or *angular frequency)* gives the number of radians per second

Motion as a Function of Time

Use of a *reference circle* allows a description of the motion

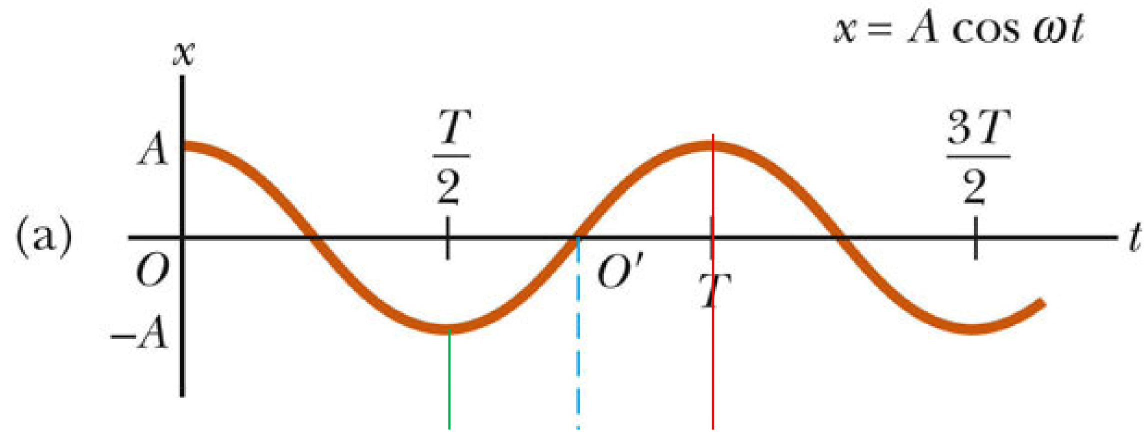
x = A cos (2πft) = A cos (ωt)

- x is the position at time t
- x varies between +A and -A



How could we plot velocity vs. time?

Graphical Representation of Motion

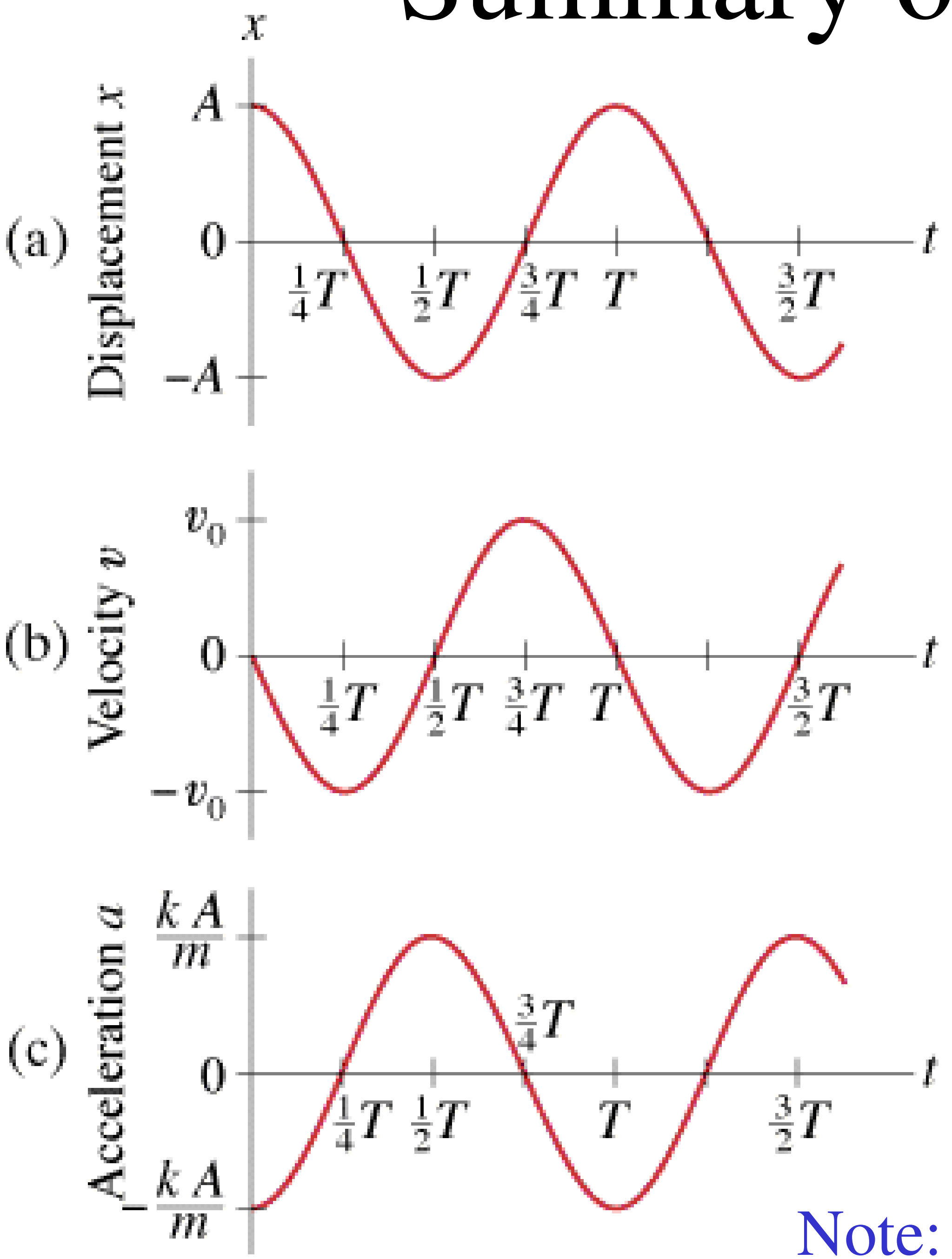


When x is a maximum or minimum, velocity is zero

When x is zero, the speed is a maximum (slope of x)

Acceleration vs. time is the slope the of velocity graph. When x is max in the positive direction, a is max in the negative direction

Summary of Formulas



$$\boxed{\omega = 2\pi f = \sqrt{\frac{k}{m}}}$$

$$x = A\cos\omega t$$

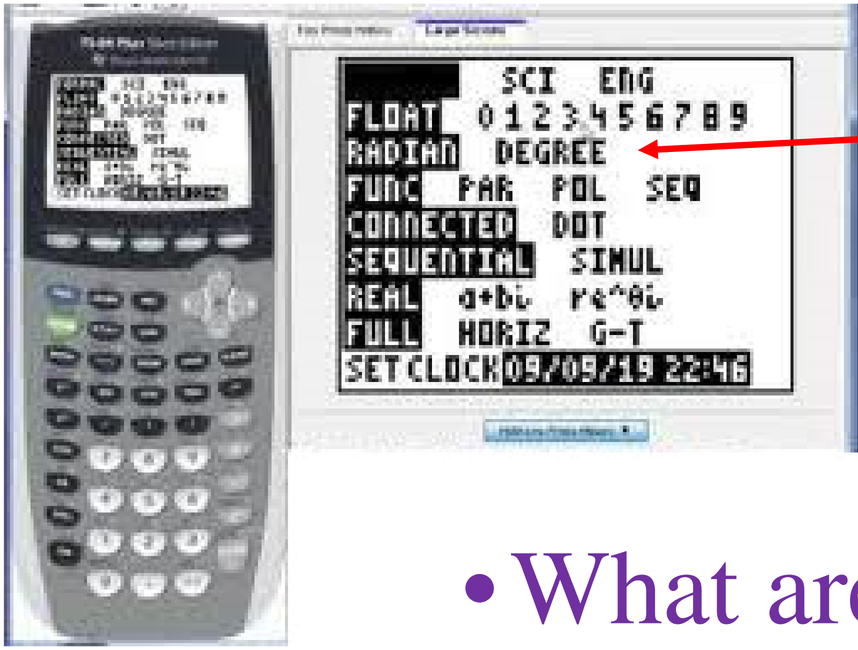
$$v = -A\omega\sin\omega t$$

$$a = -A\omega^2\cos\omega t$$

$$\boxed{a_{\max} = \frac{kA}{m}}$$

Note: v is slope of x vs. t
a is slope of v vs. t

Calculator Warning!



$$x = A\cos\omega t$$

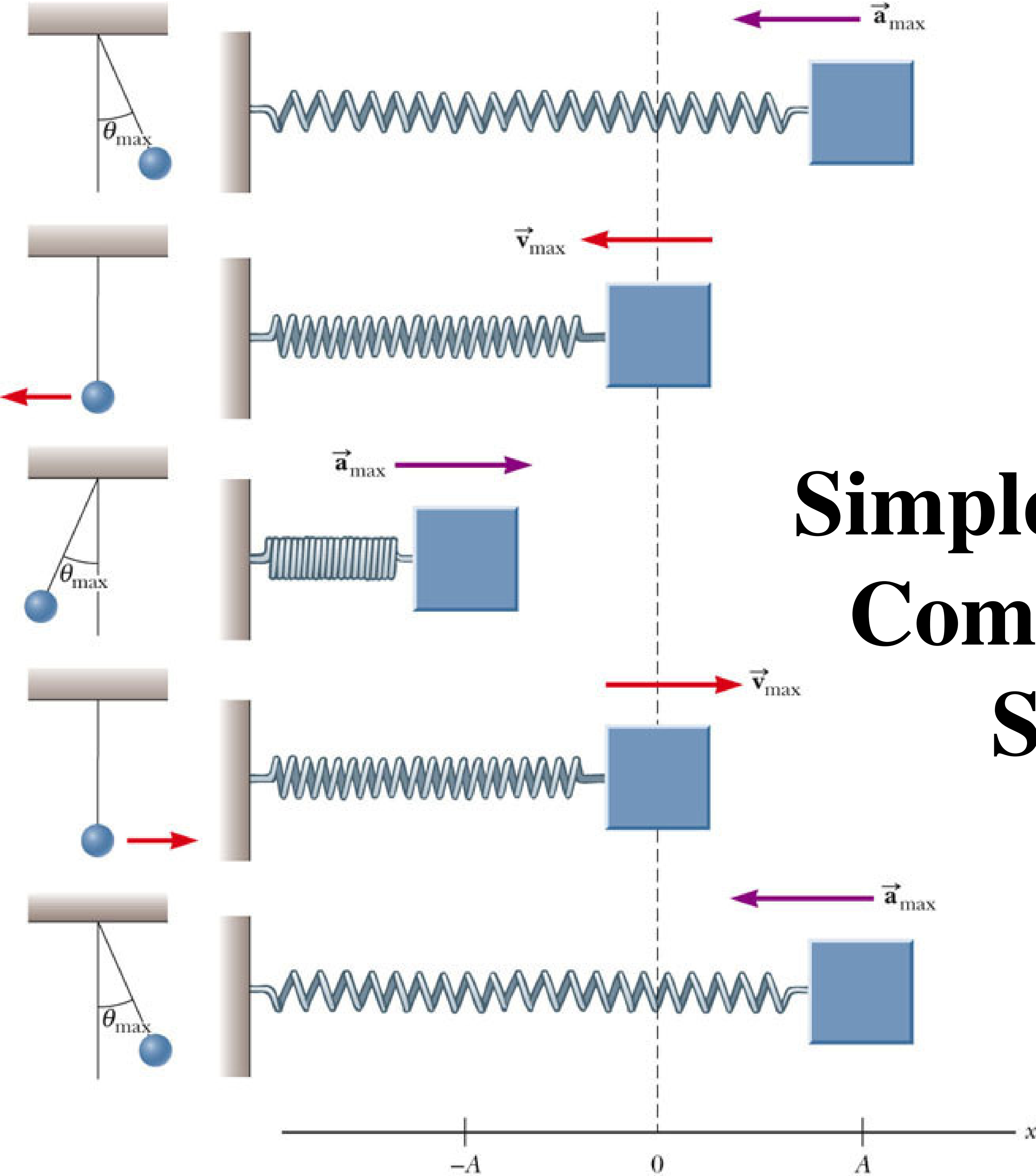
$$v = -A\omega\sin\omega t$$

$$a = -A\omega^2\cos\omega t$$

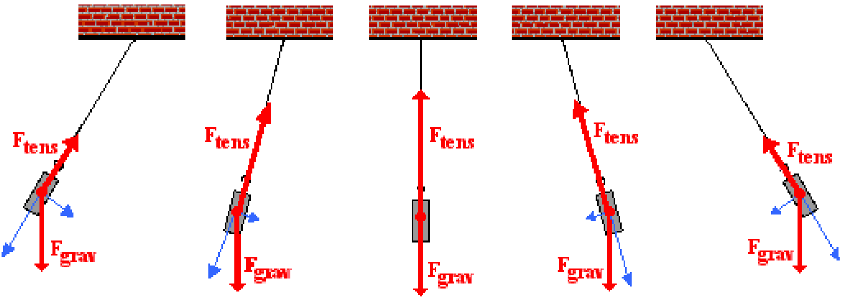
- What are the units of ωt ?

Thus, your calculator will either need to be in radians to give the correct answer, or you need to convert ωt to degrees.

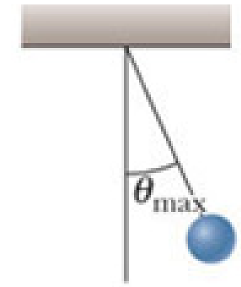
2π radians = 360°



Simple Pendulum Compared to a Springs

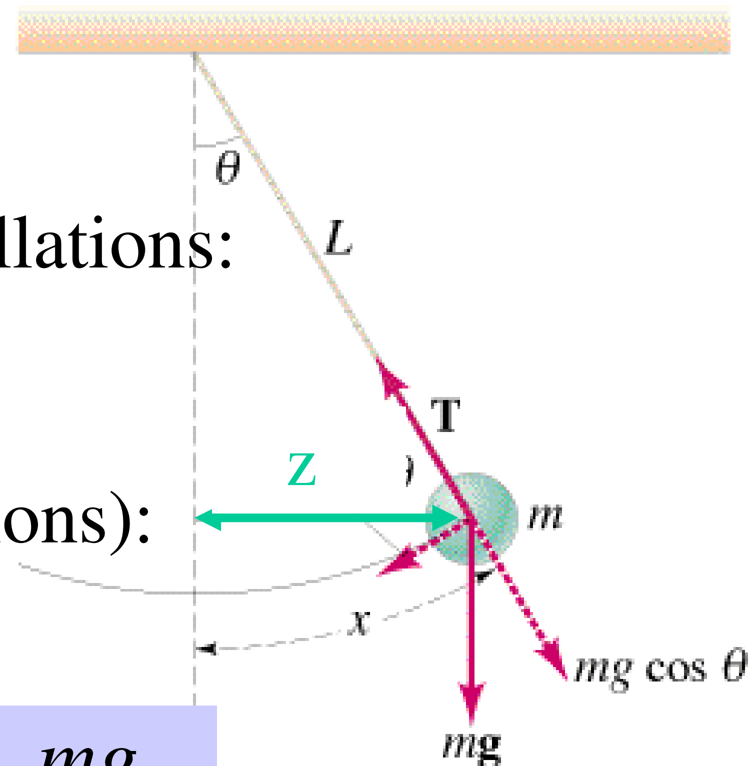


Simple Pendulum



- The simple pendulum is an example of simple harmonic motion
- Consists of small object suspended from the end of a cord. Assumptions:
  - Cord doesn't stretch
  - Mass of cord is negligible

What causes it to swing back and forth?



Gravity causes restoring force for oscillations:

$$F = -mg \sin \theta$$

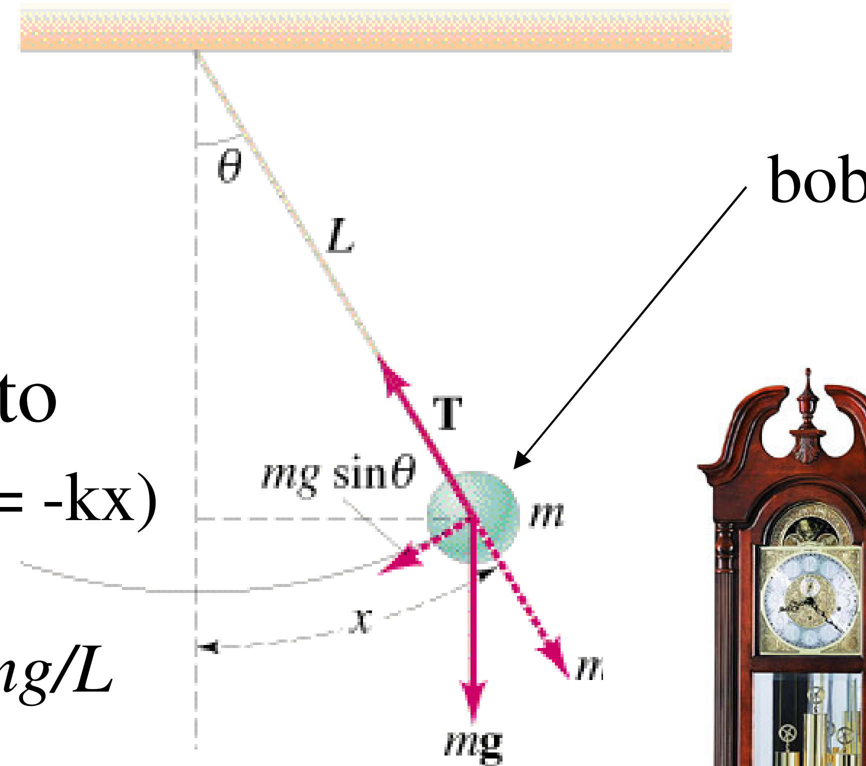
If θ is small (small amplitude oscillations):

$$\sin\theta = \frac{z}{L} \approx \frac{x}{L}$$

$$F_{pendulum} = -\frac{mg}{L}x$$

Pendulum = Simple Harmonic Motion

$$F_{pendulum} = -\frac{mg}{L}x$$



Restoring force is proportional to negative of displacement (F_{spring}= -kx)

Effective "spring constant" is $k_{eff} = mg/L$

$$T_{spring} = 2\pi\sqrt{\frac{m}{k_{eff}}} \Longrightarrow T_{pendulum} = 2\pi\sqrt{\frac{L}{g}}$$

Period of simple pendulum is independent of mass or amplitude; instead depends on the length of cord

For a pendulum clock, the timing mechanism is designed by adjusting *L*

A person swings on a swing. When the person sits still, the swing oscillates back and forth at its natural frequency. If, instead, two people sit on the swing, the natural frequency of the swing is

A. greater.
B. the same.
C. smaller.

Period of simple pendulum is independent of mass or amplitude; depends on length of cord

Q144

A spring stretches 3.0 cm from its relaxed length when a force of 7.5 N is applied.

A 0.5 kg object rests on a frictionless horizontal surface and is attached to the free end of the previously mentioned spring. The object is stretched $x = 5$ cm and released from rest at $t=0$.

a) What is the force constant of the spring?

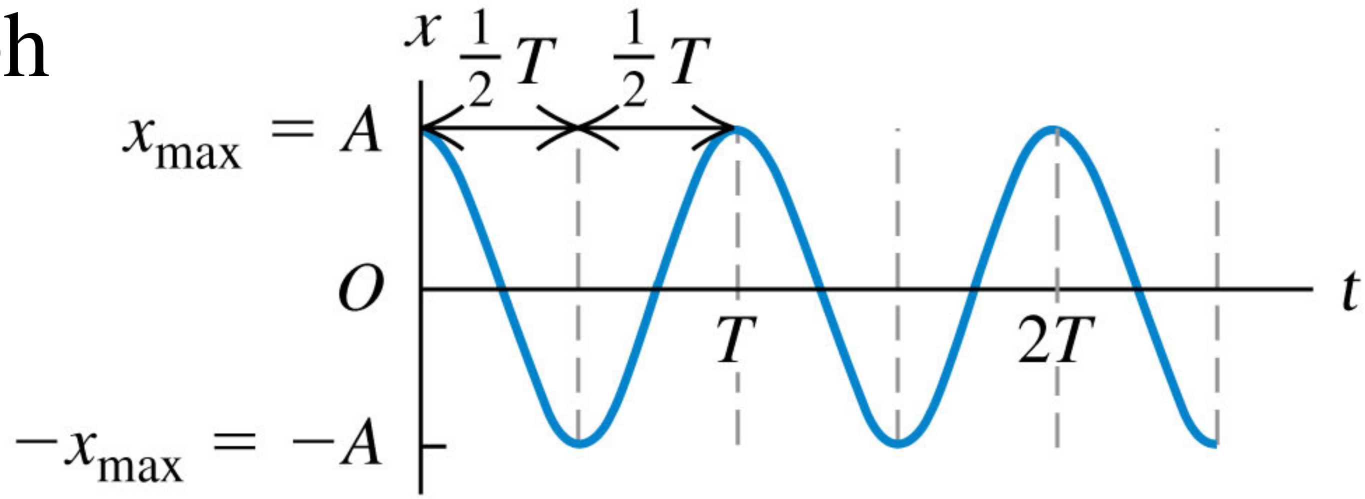
b) What are the angular frequency, frequency, and period?

c) What is the total energy of the system?

d) What are the max velocity and acceleration?

e) Find the displacement, velocity and acceleration at $t=0.5$ s.

This is an *x*-*t* graph for an object in simple harmonic motion.



At which of the following times does the object have the *most negative velocity* v_x?

A. $t = T/4$

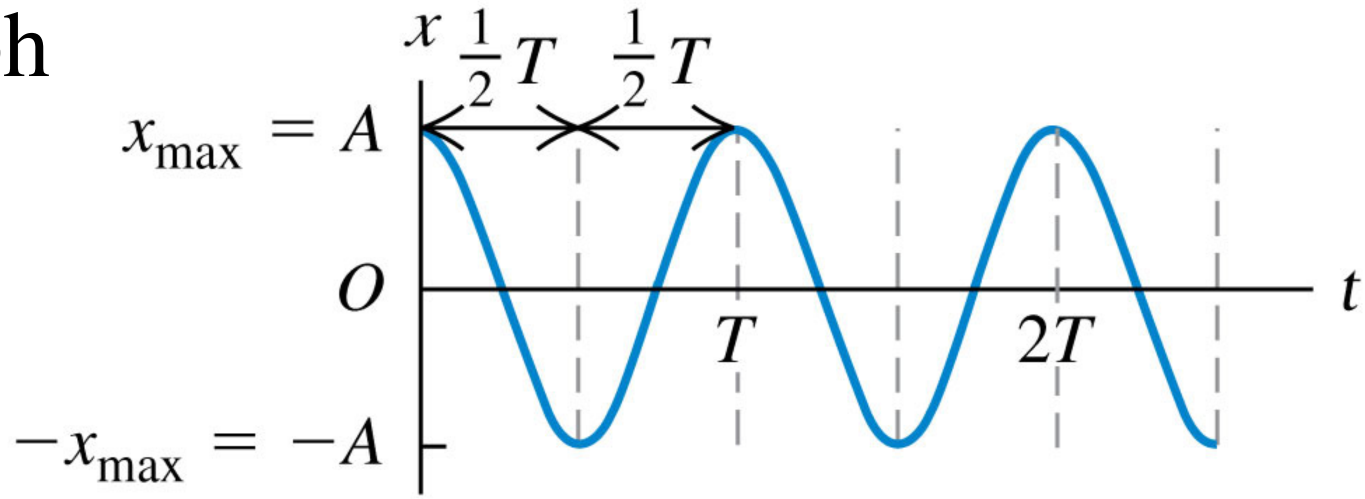
B. $t = T/2$

C. $t = 3T/4$

D. $t = T$

Q146

This is an *x*-*t* graph for an object in simple harmonic motion.



At which of the following times is the *kinetic energy* of the object the greatest?

A. $t = T/8$

B. $t = T/4$

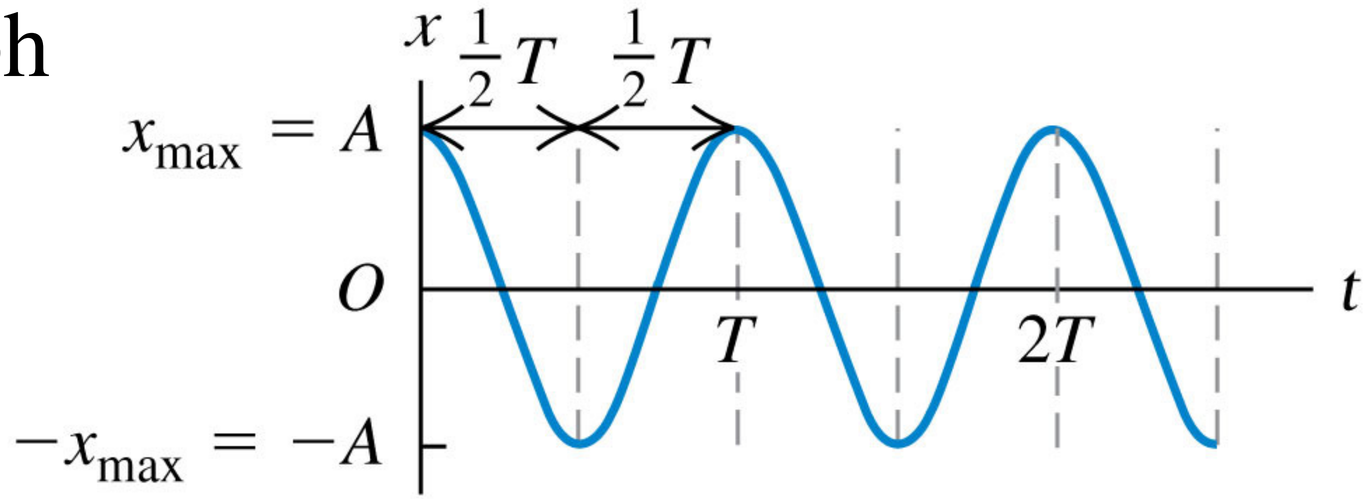
C. $t = 3T/8$

D. $t = T/2$

E. more than one of the above

Q147

This is an x-t graph for an object in simple harmonic motion.



At which of the following times is the *potential energy* of the spring the greatest?

A. $t = T/8$

B. $t = T/4$

C. $t = 3T/8$

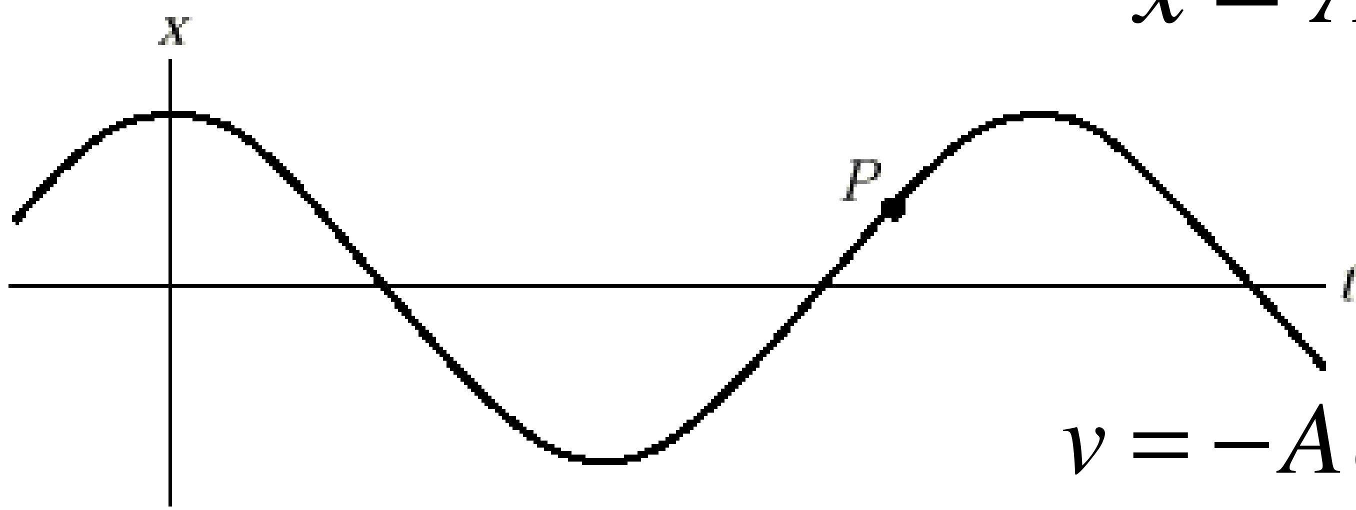
D. $t = T/2$

E. more than one of the above

Q148

A mass attached to a spring oscillates back and forth as indicated in the position vs. time plot below. At point P, the mass has

$$x = A\cos\omega t$$



$$v = -A\omega\sin\omega t$$

$$a = -A\omega^2\cos\omega t$$

A. positive velocity and positive acceleration.

B. positive velocity and negative acceleration.

C. positive velocity and zero acceleration.

D. negative velocity and positive acceleration.

E. negative velocity and negative acceleration.

Q142

A simple pendulum has mass 2 kg and length 1 m. What is the period of the pendulum?

For SHO:

$$T = 2\pi\sqrt{\frac{m}{k_{eff}}}$$

$$F = -mg \sin \theta$$

$$\sin\theta = \frac{z}{L} \approx \frac{x}{L}$$

A) 2.0 s

B) 2.8 s

C) 4.4 s

D) 8.9 s

E) 19.7 s

Q143

A person swings on a swing. When the person sits still, the swing oscillates back and forth at its natural frequency. If, instead, the person stands on the swing, the natural frequency of the swing is

A. greater.
B. the same.
C. smaller.

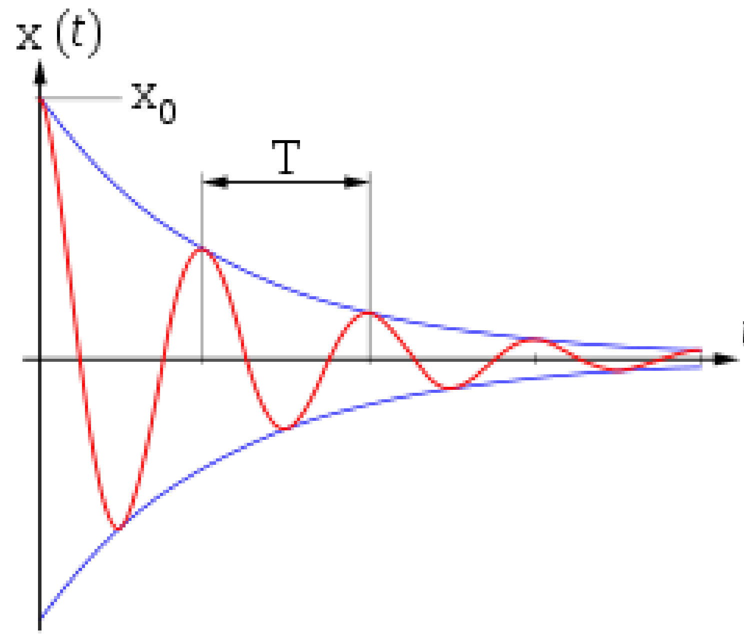
Q145

Damped Oscillations

Why does a child stop swinging if not continuously pushed?

When work is done by a dissipative force (friction or air resistance), not all of the mechanical energy is conserved.

This means not all of her potential energy at the top of each swing is converted into kinetic energy so her next swing is not as high.



The period of oscillations stays the same.
The amplitude decreases with time.

Chapter/Section: Clicker #=Answer

142=B, 143=A, 144=B, 145=A, 146=A, 147=B, 148=D